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Digital educational system design for individual work on mathematical errors of primary school students

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Abstract

Context and relevance. Currently, the possibilities and limitations of using digital technologies, including artificial intelligence, in education are being actively discussed and researched. One of the important areas of research in this field is the development of digital tools for individualization (personalization) of the educational process in schools. **Objective.** The goal is to justify and develop a digital educational system for individual work with primary school students on their mathematical errors. **Hypothesis.** The use of a digital educational system in the learning process, which allows primary school students to monitor the process of completing a mathematical task, will allow for the prompt identification and correction of possible errors, improve the quality of subject matter acquisition, and promote the development of interest in mathematical knowledge. **Methods and materials.** The pedagogical foundation of the digital educational system being developed is based on research into the phenomenon of mathematical errors, identifying their types, their causes, and methods for eliminating them. The psychological foundation of the design is based on L.S. Vygotsky's theory of the zone of proximal development and J. Bruner's concept of scaffolding. **Results.** The concept of the proposed digital educational system, "Smart Notebook," is described as follows: it will be able to analyze students' progress in solving math problems, detect errors, compare them with a database of common mistakes, identify its causes, and tailor hints and supporting tasks to each student's needs to eliminate these causes. The Smart Notebook will consider students' abilities by offering tasks within their zone of proximal development. A peer, a virtual assistant, will aid the student according to a pre-planned scenario, with various support options. **Conclusions.** The digital educational system can be used in pedagogical education to prepare primary school teachers to work on subject-specific mathematical errors.

Keywords: individualization of learning, zone of proximal development, digital educational system, smart notebook, primary school students

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Проектирование цифровой образовательной системы индивидуальной работы над математическими ошибками младших школьников

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Резюме

Контекст и актуальность. В настоящее время активно обсуждаются и исследуются возможности и ограничения использования цифровых технологий, в том числе искусственного интеллекта, в образовании. Одним из важных направлений исследований в данной области выступает разработка цифровых средств индивидуализации (персонализации) образовательного процесса в школе. **Цель.** Обосновать и разработать цифровую образовательную систему индивидуальной работы над математическими ошибками младших школьников. **Гипотеза.** Использование в учебном процессе цифровой образовательной системы, обеспечивающей младшим школьникам возможность контролировать процесс выполнения математического задания, позволит оперативно выявлять и исправлять возможные допущенные ошибки, повысит качество усвоения предметного материала, будет способствовать развитию интереса к математическому знанию. **Методы и материалы**. С педагогической позиции теоретическую основу разрабатываемой цифровой образовательной системы составляют исследования феномена математических ошибок, выделения их типов, причин появления, методики работы по их устранению. С психологической позиции теоретическую основу проектирования составляют учение Л.С. Выготского о зоне ближайшего развития и концепция Дж. Брунера о скаффолдинге. **Результаты**. Описана концепция проектируемой цифровой образовательной системы «Умная тетрадь». Этот инструмент будет способен провести анализ процесса решения обучающимся математического задания, обнаружить допущенные ошибки, соотнести их с базой ошибок, выявить причины ошибок, подобрать для конкретного обучающегося подсказки и вспомогательные задания, позволяющие устранить причины допущенной ошибки. Умная тетрадь будет учитывать возможности ученика, предлагая ему задания в зоне ближайшего развития. Помогать ученику будет сверстник — виртуальный ассистент — по запланированному сценарию с вариантами помощи. **Выводы**. Цифровая образовательная система может быть использована в педагогическом образовании при подготовке учителя начальных классов к работе над предметными математическими ошибками.

Ключевые слова: индивидуализация обучения, зона ближайшего развития, цифровая образовательная система, умная тетрадь, младшие школьники

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Introduction

In the current situation of the country's development, the role of mathematical and natural science education in ensuring technological sovereignty is increasing. However, an analysis of primary general education practices shows that a significant proportion of students experience difficulties in mastering the subject area of ‘Mathematics and Informatics’ and in developing scientific literacy (Margolis, 2021). Many schoolchildren often lose interest in studying mathematics and natural sciences at the very beginning of their education (Council Meeting..., 2025), which leads to a decline in the quality of education (Isaev, Margolis, 2023; Working with Children..., 2024). The importance of this problem for the state and plans to solve it were evidenced by the Russian Federation Government's approval of a comprehensive plan of measures to improve the quality of mathematics and natural science education until 2030 (Comprehensive Plan of Measures..., 2024).

One important way to address difficulties in learning mathematics and low motivation among students is to develop tools for primary school pupils to work on their mistakes individually. For primary school teachers, the ability to quickly diagnose learning difficulties in specific students, identify their causes, and work to eliminate them is becoming an essential component of professional competence (Isaev, Margolis, Safronova, 2023). The importance of individual work on mathematical errors lies in the fact that such work allows not only to correct mistakes, but also to form a deeper understanding of the subject being studied in students. Systematic analysis of errors, classification and development of strategies to eliminate them can significantly improve the quality of mathematics education.

However, in the traditional classroom system, where frontal teaching methods dominate, it is extremely difficult, if not impossible, for teachers to work with students individually. A common method used by teachers to deal with mathematical errors is to have students revisit the topic without identifying the causes of their mistakes.

Approaches to individualisation, differentiation, and personalisation of the educational process have been proposed over a long period of psychological and pedagogical research in Russia and abroad (Sirotyuk, 2004; Bernacki, Greene, Lobczowski, 2021).

The range of individual characteristics that should be considered when organising training is quite broad (needs, interests, personal experience, learning style, etc.), as are the aspects of the educational process that need to be worked out in order to take them into account (pace, sequence of training, content of assignments, methods of assessment, support, etc.).

One of the leading areas of individualisation (personalisation) is the development of adaptive learning technologies that use data on learner characteristics, including prior knowledge, errors and strategies used during task completion, etc., to dynamically adapt learning material and forms of interaction with learners (Aleven et al., 2017; Martin et al., 2020).

In recent decades, the prospects for implementing adaptive learning have been linked to the use of digital learning technologies, including artificial intelligence (Gligorea et al., 2023; Tan et al., 2025). Significantly, adaptive tools demonstrate particular potential in improving the quality of learning in subjects with a formalised structure (mathematics, basic scientific literacy).

An important and growing area of research that requires further attention is the use of digital adaptive learning systems to support self-regulated learning (Khalil et al., 2024). Knowledge tracing is the use of data from the analysis of learners' learning interactions (exercise completion, task responses) to assess their knowledge status (i.e., unexplored and learned) (Liu, 2022; Shen et al., 2024).

One of the most promising areas of scientific research and applied solutions is the analysis of student errors while completing tasks, followed by the selection of tasks (exercises, prompts). A significant number of such solutions of varying types and scales are already being implemented in widespread educational practice abroad. These solutions diagnose errors, generate prompts, and select personalized tasks and exercises (ALEKS, ASSISTments, DreamBox).

The widely popular Khan Academy is an educational platform that features challenges, educational videos for different user roles — student, teacher, parent — and step-by-step hints that guide students to the answer after making a mistake.

In Russia, such solutions have also become widespread in the rapidly growing online learning sector. One of the most interesting examples is Yandex Textbook, a domestic service offering lessons for elementary and middle school students with automatic answer checking. If you make an incorrect answer, you can open a hint and solve the problem again. Hints can contain step-by-step explanations; for example, after three incorrect answers, you can move on or continue solving the problem with the hint. When solving problems, a character accompanies the text and "reacts" to the student's correct and incorrect solutions.

The potential and limitations of educational and technological solutions in this area are the focus of research (Stephens-Martinez and Fox, 2018; Munshi et al., 2023; Jangra et al., 2025; Zerkouk et al., 2025). Developing methodological and technological solutions that would allow for the typology of student errors and the selection of tasks and prompts to eliminate the cause of the error remains a pressing issue.

The purpose of the study presented here was to provide a psychological and pedagogical justification and develop a digital educational system for individualized work on mathematical errors in primary school children. This system is capable of working with all students within and outside of class by providing real-time feedback during independent task completion. The concept is to design a digital system that is grounded in theoretical ideas from cultural-historical psychology and activity theory and that utilizes the ability to construct a child's zone of proximal development (ZPD) when solving mathematical problems in a digital environment. The digital system will serve as a space in which the ZPD is constructed through prompts that correlate with types of assistance (scaffolding) and guide the child step-by-step through the task. The virtual assistant will serve as a peer, a "smarter companion," as defined by L.S. Vygotsky.

The design of the digital educational system is based on the following **hypothesis**: the use of a digital educational system in the educational process, which provides primary school students with the opportunity to control the process of completing mathematical assignments, will allow for the prompt identification and correction of possible errors, improve the quality of assimilation of subject material, and will promote the development of interest in mathematical knowledge and subject-related thinking.

Materials and methods

From a pedagogical perspective, the theoretical basis for the digital educational system being developed for individualized work on mathematical errors is based on research into the phenomenon of errors, identifying their types, their causes, and methods for eliminating them. Research on mathematical errors has a long tradition in global pedagogical, psychological, and methodological literature (Sanina, Sokolov, 2021; Sokolov, 2023).

Of fundamental importance for the digital system for individual work on mathematical errors being developed is the inclusion of their typology developed by R. Ashlock et al. (Ashlock, 2010; Radatz, 1979; Brown et al., 2016; Watson et al., 2018; Kakoma et al., 2021; Fiori et al., 2025), which represents a systematic approach to classifying errors made by students when learning a new topic in mathematics. Understanding these errors helps teachers develop more effective teaching strategies and adjust teaching methods. Their analysis is an important stage in the teacher's work, allowing them to identify not only systematic errors but also students' misconceptions about mathematical concepts. This stage can be used to develop diagnostic tools to help teachers identify gaps in the knowledge of younger students. The typology mentioned above can be adapted for different levels of education and different areas of mathematics.

When developing a digital educational system for individualized work on mathematical errors in primary general education, two types of errors were considered: conceptual and procedural errors. The former are students' misunderstanding of basic concepts or the incorrect application of concepts and rules. The latter are errors in the use of mathematical formulas. The primary focus of the digital system on correcting mathematical errors is on the first type.

From a psychological perspective, the theoretical basis of the developed digital educational system for individual work on mathematical errors is formed by L.S. Vygotsky's teaching on the zone of proximal development (Vygotsky, 1991; Margolis, 2020) and J. Bruner's concept of scaffolding (Margolis, 2020).

In this article, I.A. Kotlyar and M.A. Safronova present an analysis of three concepts from cultural-historical psychology and the cultural-historical theory of activity: zone of proximal development, learnability, and scaffolding. These concepts describe the reality of child development as it interacts with adults. The relationships between these three concepts and the scope of their application to solving learning problems are described. Scaffolding is an adult's actions toward a child (Wood, 1976), constructing the child's ZPD space. Learnability is a child's ability to learn new things by advancing within their zone of proximal development with the help of an adult, within the scaffolding they construct. Learnability is an important characteristic of a child's actual development (Kotlyar, Safronova, 2011).

The authors understand the complexity of transferring communication and interaction processes to a digital environment (not the full spectrum of interaction can be replicated in a digital system). However, since the Smart Notebook is designed as a teaching tool, live interaction between the student and teacher is not replaced by working with it. This tool is initially designed as a way to individually assist young students in solving mathematical problems and mastering essential arithmetic operations, as a space built specifically for them and within their zone of proximal development.

The "Smart Notebook" digital system (web application) will address the issue of individualizing the learning process for primary school students when working on mathematical errors in their subjects. It will enable prompt monitoring of the problem-solving process and conduct individualized developmental assessments not only after the problem-solving process is completed but also while it is ongoing, opening up significant potential for improving the quality of education.

Results

The pilot version (prototype) of the "Smart Notebook" digital system (see Fig. 1), being developed at Moscow State Pedagogical University, is based on the "Subtraction" topic in the primary school curriculum. Arithmetic operations are traditionally a core component of the "Mathematics" course in primary general education. The topic of "Subtraction" is an integral part of the mathematics curriculum (On Approval of the Federal Law, 2021; On Approval of the Federal Law, 2023). Upon completion of this topic, primary school graduates should be able to perform written subtraction with multi-digit numbers up to a million using addition tables and a written subtraction algorithm.

Teaching written calculations (column) should be based on a well-established understanding of place values and mental calculation techniques. The transition to written calculations, including written subtraction, should be subject-based, accompanied by case studies

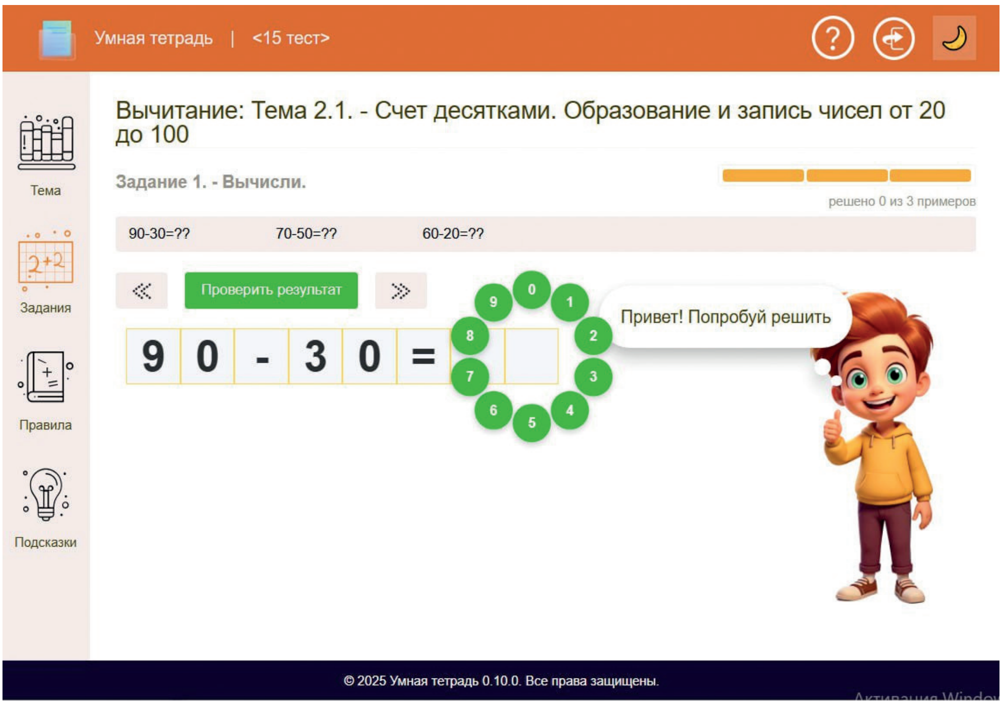

Fig. 1. Screenshot of the “Smart Notebook” web app prototype

to develop meaningful generalizations and the construction of an algorithm for performing the action, involving the students themselves. If the focus of teaching written subtraction is on developing the skill of step-by-step execution of the algorithm without establishing an adequate connection between its steps and the mathematical essence of the actions being performed, numerous errors will inevitably arise. In other words, it is necessary to ensure an understanding of the essence of the algorithm, which provides the key to its meaningful application.

We've broken the topic of "Subtraction" into teaching units that correspond to traditional approaches to teaching the arithmetic component of a primary general education mathematics course. By a teaching unit, we mean a portion of the educational material that represents a coherent element in its semantic meaning and fulfills a specific function in achieving the stated objectives in mastering the subject.

When designing assignments for each teaching unit, we followed general rules. Assignments are presented to students sequentially, in order of increasing difficulty. Below is a brief description of each assignment.

Task 1 is a traditional, basic-level task that directly applies the learned calculation technique. It requires the student to directly apply a rule. However, Task 1 is important for diagnosing any deficiencies in the student's skills. Depending on the error made during the task, it can be assumed that a particular difficulty is associated with a gap in mastering one of the previous topics. In some cases, typically when the calculation technique involves multiple operations, Task 1 is divided into two stages. First, Task 1.1 is presented, in which the student must perform the operation in its expanded form, demonstrating the ability to correctly perform each individual operation. Then, Task 1.2 is presented, in which the student must perform the operation in its collapsed form. We consider both stages to be a single, basic-level Task 1.

Task 2 requires the student to apply the learned rule in a slightly modified situation. The task may require the student to perform a reverse action instead of a forward one. For example, the basic-level task "Calculate 40 — 7" could be transformed into "Insert the missing number so that the equation becomes true: 40 — ☐ = 33 or ☐ — 7 = 33." Task 2 may ask the student to find a simple pattern in a series of problems to solve. We assume that the student who completed Task 1 was able to sufficiently address their existing skill gaps. Therefore, Task 2 is less significant in terms of identifying the student's skill deficits. Errors in this task are likely related to an insufficiently developed skill in using the learned calculation method. If errors in the task are not corrected, a hint is offered first, followed in some cases by the correct answer or solution, and in others by a transition to help.

Task 3 requires the student to demonstrate generalization of the calculation technique being mastered. This could be a task involving "fairytale" numbers, a task requiring reconstruction of the learned method of operation, or the recognition of a more complex pattern in a series of solved examples. If the student cannot solve Task 3 or solves it incorrectly, they are not demonstrating generalization. In this case, they are first offered a hint, and then the correct answer to the task or its solution.

When completing tasks, children will inevitably make mistakes. For each task, we describe the most likely types of errors. Depending on the error the child made, we analyze the solution process — establishing a connection between the error and the skill gap that led to it, thus identifying the causes of the error.

The scaffolding process for student interaction with the digital system occurs as follows. If a child makes a predictable error, they are directed to practice it in one of the previous topics. If this is the first task in which we test basic computational skills, we direct the student to the topic in which we find a gap. If it is a more complex task, we first provide a hint, and if the hint fails, we provide the correct solution. This type of support for students interacting with the Smart Notebook distinguishes it from existing digital educational systems.

We examined the main student errors in the "Mathematics" course for primary general education, using the topic "Subtraction" as an example in our article (Sokolov, 2023). The "Smart Notebook" contains a system of hints and assistance for students depending on the difficulty level of the task.

The first type of assistance for completing a Basic Level 1 task is that after entering the answers to the task, which typically contains several individual examples, the student receives feedback indicating which examples were answered correctly and which were incorrect. If there are any errors, they are prompted to correct them. If there was only one error and it is corrected, we assume the error is random, and the student can move on to the next tasks.

If there were more errors and they were corrected, then the assistance consists of asking the student to repeat a similar task in order to develop a more stable skill in using the calculation technique being mastered.

If a student is unable to independently correct errors in a task, they are asked to perform the error using a dynamic numerical model. A dynamic numerical model is a software feature that presents the student with a visual object model of a given number, in the form of a visual representation of the number of place values. The model allows the student to interact with the visual object model, i.e., perform an arithmetic operation based on its object-based basis. If this type of prompting fails to correct the error, assistance is provided in the form of a detailed description of the rule underlying the arithmetic operation. The rule is accompanied by an example and, in most cases, a test task, the completion of which will confirm the student's understanding of the calculation procedure. After this type of assistance, the student is again asked to correct the errors in the task and then repeat the same task.

In cases where a student makes an error that is in the register of possible errors, he is directed to the topic corresponding to the error he made.

The first type of assistance for more complex tasks 2 and 3 is similar to that for task 1: the student is given information about correct and incorrect answers and asked to correct them. Since more complex tasks are offered to students after successfully completing the basic task, we believe it is inappropriate to use assistance in the form of presenting a rule. In some cases, additional skill gaps may be discovered during task 2 that were not identified in task 1. In such cases, the student is directed to the topic corresponding to the gap. In other cases, the student is first offered a hint and then the correct answer to the task or its solution.

The Smart Notebook task support system we've described allows students to complete tasks within their ability, while also highlighting any errors they've made, ensuring they're corrected, or, in some cases, demonstrating the correct solution. In other words, the digital system ensures students learn within their zone of proximal development.

Let's consider the capabilities of the "Smart Notebook" digital system to analyze the problem-solving process, identify the causes of errors, and select hints and additional tasks to fill skill gaps. We will use the example of a student completing the second-grade mathematics course unit "Calculation Techniques for Cases of the Type 60–24." To successfully complete the assignments on this topic, the student must first possess the skills developed during the study of several previous topics: Topic 1 "Counting by Tens. Subtraction of the Type 50–20"; Topic 2 "Subtraction of the Type 35–30, 35–5" (based on representing a number as a sum of place value addends); Topic 5 "Calculation Techniques for Cases of the Type 30–7".

Task 1.1.

Write the calculations using the example:

90 – 43 = (90 – 40) – 3 = 50 – 3 = 47:

70 – 28 = (70 – ☐) – ☐ = ☐ – ☐ = ☐

60 – 36 = (60 – ☐) – ☐ = ☐ – ☐ = ☐

Solution:

70 – 28 = (70 – 20) – 8 = 50 – 8 = 42

60 – 36 = (60 – 30) – 6 = 30 – 6 = 24

As we can see, the correct execution of the calculation technique consists of several operations: 1) represent the number 28 as the sum of the place value addends 20 + 8 (a skill developed in Topic 2); 2) perform the subtraction 70 – 20 (Topic 1); 3) perform the subtraction 50 – 8 (Topic 5). In completing task 1.1, the student must perform the operation in expanded form and demonstrate the ability to correctly perform each of the operations.

Let's consider possible errors in completing task 1.1. In compiling the error list, we relied on R. Ashlock's typology of mathematical errors and on the results of testing the tasks on elementary school students.

1) Conceptual error — the inability to factor a two-digit number into the sum of its place-value addends.

For example, 70 – 28 = (70 – 2) – 8.

Any numbers other than the correct ones (20 and 8) will indicate this error in the solution record. If the error is not corrected after the digital system reports an incorrect operation, the student will be referred to Topic 2 to correct the identified gap.

2) Conceptual error in subtraction of the form 50 – 20.

Например, 70 – 28 = (70 – 20) – 8 = 68 – 8.

For example, 70 – 28 = (70 – 20) – 8 = 68 – 8.

A marker for this error will be any number in the solution except the correct one: 50. All numbers written to the left of this number are correct. If the error is not corrected after the digital system reports an incorrect operation, the student will be referred to Topic 1 to address the identified gap.

3) Conceptual error. Errors in subtraction of the type 50 – 8.

For example, 70 – 28 = (70 – 20) – 8 = 50 – 8 = 30.

The error marker will be any number in the solution except the correct one: 42. All numbers written to the left of this number are correct. If the error is not corrected after the digital system reports an incorrect operation, the student will be referred to Topic 5.

After working through all the identified gaps, the student still cannot correct their errors, they will be prompted to perform an action using a dynamic number model. An image of the number 70 will appear, represented by 7 tens, from which the student must interactively subtract 2 tens and 8 ones.

Finally, the last type of help will be a detailed description of the rule for subtracting one round two-digit number from another two-digit number, also containing an example and a test task.

After all the work has been completed, the student will be asked to repeat task 1.1.

Write the calculations using the example.

90 – 43 = (90 – 40) – 3 = 50 – 3 = 47

80 – 42 = (80 – ☐) – ☐ = ☐ – ☐ = ☐

50 – 27 = (50 – ☐) – ☐ = ☐ – ☐ = ☐

Task 1.2.

Calculate:

80 – 47 = ☐

70 – 32 = ☐

60 – 56 = ☐

100 – 24 = ☐

In Task 1.2, the student must perform an operation in a reduced form.

Let's look at possible errors in completing Task 1.2.

1) Errors in knowledge of the addition table:

For example, 80 — 47 = 34 or 80 — 47 = 23.

We assume that the most common manifestation of a computational error is a discrepancy between the entered answer and the correct one by one. In this example, this would be 10 – 7 = 4 or 8 – 4 = 3. In such cases, we assume a computational error and send the student to the number composition simulator, where they will be presented with several tasks of the following type: 10 = 8 + ☐, 10 = ☐ + 3 (the composition of the number 10), or 8 = 4 + ☐, 8 = ☐ + 2 (the composition of the number 8).

2) Procedural error: 80 – 47 = 47.

The child calculates: 80 – 47 = 80 – 40 + 7 = 47.

The student makes an error in subtracting two-digit numbers from one round number: instead of subtracting 7, they add it. This error is related to the current teaching unit; the student will be asked to correct it using the dynamic number model.

If Task 1.2 contained errors, the student will be asked to repeat Task 1.2 after correcting them.

Calculate:

70 – 36 = ☐
60 – 43 = ☐
80 – 74 = ☐
100 – 52 = ☐

Task 2.

Guess the rule used to create Table 1.

Using this rule, fill in the blanks with numbers.

Table 1

Assignment for the student

Minuend	90	80	70		50		
Subtrahend	68	57	46	35			
Difference				25	26		

Answer (table 2):

Table 2

Answer

Minuend	90	80	70	60	50	40	30
Subtrahend	68	57	46	35	24	13	2
Difference	22	23	24	25	26	27	28

This task requires the student to perform not only the direct operation but also the inverse: find the minuend, find the subtrahend, and establish a simple pattern in a series of given examples. In this case, each successive minuend decreases by 10, and the subtrahend by 11, resulting in an increase in the difference by one.

Possible errors in completing task 2.

1) Errors in knowledge of the addition table:

For example, 90 – 68 = 23 (the child calculates: 10 – 8 = 3) or 90 – 68 = 12 (the child calculates: 90 – 60 = 20).

The student will be sent to a simulator to practice the composition of the number 10 or 9.

2) All other possible errors are addressed in tasks 1.1 and 1.2. If uncorrected errors are still present in the first five columns, the student will be asked to work with a dynamic number model and presented with the rule for subtracting one round two-digit number from another two-digit number.

3) If there are uncorrected errors in the last two columns, the student is first shown a hint, then the correct solution.

Hint: "Notice how the minuend changes, and how the subtrahend changes."

Task 3.

Figure 3 shows a fragment of consecutive fairy-tale numbers:

Fig. 2. Fairy-tale numbers

Calculate:

ψ0 – 26 = ☐
ω0 – γ3 = ☐
φ0 – ω4 = ☐

Solution:

ψ0 – 26 = β4
ω0 – γ3 = 7
φ0 – ω4 = 16

"Fairytale" numbers are a technique often found in mathematics textbooks based on the D.B. Elkonin-V.V. Davydov system (Aleksandrova, 2023; Davydov, 1996). They allow one to assess whether a student has mastered a particular method of operation at the level of substantive generalization. Problems of this type are undoubtedly of a higher level of difficulty. Solving problems with "fairytale" numbers allows students to advance their educational trajectory, as they offer the opportunity to discover patterns hidden when performing arithmetic operations with ordinary numbers. In this topic, such a generalization is the fact that the number of tens of the difference will always be n + 1 less than the number of tens of the minuend, where n is the number of tens of the subtrahend. If a student gives an incorrect answer, they are offered a hint and then the correct solution.

Clue:

«$\psi0 - 26 = \psi0 - 20 - 6 = \gamma0 - 6 = \cdots$

The digit ψ is 2 times greater than the digit γ, so ψ tens minus 2 tens is γ tens.

Continue solving the problem yourself."

Discussion of results

We have described the concept of the planned digital educational system "Smart Notebook." It can be concluded that it will be able to analyze the student's process of solving a math problem, detect errors, compare them with a database of errors, identify the causes of the errors, and tailor hints and auxiliary tasks to the specific student to eliminate the causes of the error.

The smart notebook will take the student's abilities into account, offering them tasks within their zone of proximal development. A peer — a virtual assistant — will assist the student according to a pre-planned scenario with support options. If the child consistently experiences difficulty completing complex tasks, such tasks will either not be offered in the future or will be offered based on the student's own choice.

Conclusion

1. The "Smart Notebook" digital educational system, developed as a web application, is an adaptive learning solution aimed at individualizing mathematics instruction for primary school students. The system's theoretical foundations are based on R. Ashlock's typology of mathematical errors, L.S. Vygotsky's concept of the zone of proximal development, and J. Bruner's scaffolding concept.

2. The system analyzes the problem-solving process, identifies errors, and provides assistance to students through an interactive virtual assistant. The virtual assistant offers hints to students as they solve problems. For teachers, the system provides detailed statistics for each student, including error types and progress, allowing for individualized learning paths and differentiated approaches during group work.

3. The Smart Notebook can be used in the educational process as a teacher's assistant, including during independent math assignments for primary school students. It will facilitate a differentiated approach to teaching, allowing the teacher to quickly adapt the learning process to the individual needs of primary school students in solving math problems, both during class, including dividing students into groups based on their levels, and during independent work.

4. Such a digital educational system can be used in pedagogical education when preparing primary school teachers to work on subject-specific mathematical errors.

Limitations. The limitations of the methodological, teaching, and technological aspects of the digital system — the "Smart Notebook" web application — will be determined based on the results of the ongoing pilot study in primary schools. Specifically, opportunities will be identified for identifying the most relevant topics for expanding the task banks, optimizing the system's functionality, and the types of assistance provided by the virtual assistant.

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Исаев Е.И. — аннотирование; анализ литературы; написание рукописи.

Марголис А.А. — идеи исследования; анализ литературы; написание рукописи.

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