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Modeling of dynamic systems with interval parameters. Review of methods and software tools

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The paper provides a review of existing libraries and methods of modeling dynamic systems with interval parameters. Available software libraries AWA, VNODE-LP, COZY Infinity, RiOT, FlowStar, as well as the author's adaptive interpolation algorithm are considered. The traditional software for interval analysis gives guaranteed estimates of solutions, however, over time, these estimates become extremely significantly overstated. Due to the use of a fundamentally different approach to constructing solutions, the adaptive interpolation algorithm is not subject to the accumulation of errors, determines the boundaries of solutions with controlled accuracy, and works much faster than analogues.

Keywords: interval methods, dynamic systems with interval parameters, adaptive interpolation algorithm, libraries with methods, AWA, VNODE, COSY Infinity, RiOT, FlowStar, verifyode.

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