◇◇◇◇◇◇◇◇◇◇**АНАЛИЗ ДАННЫХ**◇◇◇◇◇◇◇◇◇◇

The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application

Lev S. Kuravsky*
Moscow State University of Psychology and Education, Moscow, Russia
ORCID: https://orcid.org/0000-0002-3375-8446
e-mail: l.s.kuravsky@gmail.com

Denis A. Pominov**
Moscow State University of Psychology and Education, Moscow, Russia
ORCID: https://orcid.org/0000-0002-1321-3713
e-mail: pominovda@mgppu.ru

Grigory A. Yuryev***
Moscow State University of Psychology and Education, Moscow, Russia
ORCID: https://orcid.org/ 0000-0002-2960-6562
e-mail: g.a.yuryev@gmail.com

Nataliya E. Yuryeva****
Moscow State University of Psychology and Education, Moscow, Russia
ORCID: https://orcid.org/0000-0003-1419-876X
e-mail: yurieva.ne@gmail.com

Mariya A. Safronova*****
Moscow State University of Psychology and Education, Moscow, Russia
ORCID: https://orcid.org/0000-0002-3597-6375
e-mail: mariasaf@gmail.com

Yevgeny D. Kulanin******
Moscow state University of Psychology & Education, Moscow, Russia
ORCID: https://orcid.org/0000-0001-6093-7012
e-mail: lucas03@mail.ru

Svetlana N. Antipova *******
Moscow State University of Psychology and Education, Moscow, Russia
ORCID: https://orcid.org/0000-0001-6642-7953
e-mail: antipovasn@mgppu.ru

Presented is a mathematical model of the self-learning adaptive trainer intended for adaptive learning and providing task selection. The approach in question is an alternative to the adaptive technologies based on the Item Response Theory. Possibility

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

to take into account temporal dynamics of solution ability as well as smaller number of tasks that must be performed by a subject to provide the given results are among the features of the methods in use. To assess the effectiveness of the adaptive trainer concept under consideration, its web-implementation intended for training school students to solve mathematical tasks covered by the school curriculum was employed. The analysis performed revealed both high efficiency of the adaptive trainer (the mean test rating has increased 1.54 times owing to its use) and proven statistically significant influences of the adaptive training factor on the observed mathematical test results.

Keywords: adaptive learning, Markovian random processes, adaptive trainer, self-learning systems.

Lev S. Kuravsky, Doctor of Engineering, professor, Dean of the Computer Science Faculty, Moscow State University of Psychology and Education, Moscow, Russia, ORCID: https://orcid.org/0000-0002-3375-8446, e-mail: l.s.kuravsky@gmail.com

****Denis A. Pominov,*** Research Scholar, Computer Science Faculty, Moscow State University of Psychology and Education, Moscow, Russia, ORCID: https://orcid.org/0000-0002-1321-3713, e-mail: pominovda@mgppu.ru

*****Grigory A. Yuryev,*** PhD in Physics and Matematics, Associate Professor, Head of Scientific Laboratory, Moscow State University of Psychology and Education, Moscow, Russia, ORCID: https://orcid.org/ 0000-0002-2960-6562, e-mail: g.a.yuryev@gmail.com

******Nataliya E. Yuryeva,*** PhD in Engineering, Research Fellow, Information Technology Center for Psychological-Ecological Studies of the Faculty Newsletter-Technologies, Research Associate, Moscow State University of Psychology and Education, Moscow, Russia, ORCID: https://orcid.org/0000-0003-1419-876X, e-mail: yurieva.ne@gmail.com

*******Maria A. Safronova,*** PhD in Psychology, Dean of the Faculty of Psychology of Education, Moscow State University of Psychology and Education; Ph.D. in Psychology, Research fellow of the laboratory of Theoretical and experimental issues in cultural-historical psychology, Moscow, Russia, ORCID: https://orcid.org/0000-0002-3597-6375, e-mail: mariasaf@gmail.com

********Yevgeny D. Kulanin,*** Candidate of Physical and Mathematical Sciences, Professor, Moscow state University of Psychology & Education, Moscow, Russia, ORCID: https://orcid.org/0000-0001-6093-7012, e-mail: lucas03@mail.ru

*********Svetlana N. Antipova,*** Deputy Dean for Extra-Curricular Work of the Faculty of Information Technology, Moscow State University of Psychology and Education, Moscow, Russia, ORCID: https://orcid.org/0000-0001-6642-7953, e-mail: antipovasn@mgppu.ru

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

1. INTRODUCTION

In recent years, the popularity of e-learning that covers, in broad sense, practically all forms and ways of transmitting knowledge, abilities and skills through information and communication technologies has been increased. The approach in question has well-known advantages and disadvantages, the most significant of which is the lack of *effective adaptation of the educational process to individual characteristics and capabilities of its participants*. The problems associated with the adaptation of this kind are difficult to solve and the most topical during informal skill training, including solution of mathematical, technical, algorithmic, inventive and other problems of similar nature.

In the case of traditional teaching, these problems are usually overcome by means of interaction with a qualified teacher who taking into account known characteristics of his students, creates individual learning trajectories for them and supervises the educational process at all of its stages. Such a work, as a rule, is not amenable to automation being some type of art. It should be noted that construction of these individual trajectories requires certain forms of diagnostic solutions which clarify students' features and capabilities [19].

The problem of automating adaptive informal skill training has no satisfactory solution by now. Existing facilities of e-learning management [2, 3, 13], including learning management and content management systems, bypass this problem and solve more compliant tasks. Primarily, this situation results from both difficulties of formalization and absence of suitable mathematical tools.

Proposed in this work is the approach to solve this problem, which is based on the *adaptive testing* concept developed by the authors earlier. This concept employs trained structures represented by Markovian processes [4–12]. The approach in question is an alternative to adaptive technologies based on *the Item Response Theory (IRT)*, with the G. Rasch model being in use [1, 14–15]. This model assumes that the probability of a correct response is determined by the difference between evaluations of subject relevant ability and task difficulty. Applications of these technologies result in the following problems:
−   "Static" estimates: ignoring the fact that testing results, in general, may be changed significantly over time due to subject fatigue and other factors
−   Inability to take into account the time spent on performing tasks to calculate evaluations
−   Performing sufficiently large number of tasks to get evaluations of reasonable accuracy
−   Assessments of result reliability, which are rather complicated for practical use.

The issues presented made it topical to develop new technologies in the given field. As an approach to overcome this problem, the following is a concept of adaptive trainer based on a probabilistic model for learning informal skills, which are necessary for solving mathematical and other problems of sufficiently high complexity, with mastery in both standard techniques of constructing arguments and creative thinking being in demand. Features of the diagnostic techniques being in use to select the tasks presented are:
−   Detecting and using temporal dynamics of variable subject ability to cope with the test tasks when constructing the calculated estimates
−   Possibility to take into account the time spent for performing tasks when constructing the calculated estimates

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

- In comparison with other approaches, fewer number of presented tasks to speed up the testing process
- Advanced technique for identification of model parameters.

The given capabilities ensure advantages of the approach presented here over similar ones. It should be noted that this approach is most effective when it is necessary to organize and arrange the skills, abilities and knowledges, which are already gained (in particular, during the preparation for exams). It is less effective for the introduction to material under study.

2. THE CONCEPT OF ADAPTIVE TRAINING

When training is under way, task selection is performed with the aid of parametric mathematical models represented by *Markovian discrete-state continuous-time random processes*. Models for describing dynamics of state-to-state transitions are depicted as oriented graphs where nodes correspond to states and arcs correspond to the transitions to which the properties of Poisson flows of events are applied. In these flows, the number of events X, which fall in any time interval of length τ beginning at the time point t, is distributed according to *the Poisson Law*:

$$P_{t,\tau}\left(X=m\right)=\frac{a\left(t,\tau\right)^{m}}{m!}e^{-a(t,\tau)},$$

where $P_{t,\tau}(X=m)$ is probability of m events during the given interval, $a(t,\tau)$ is the mean of events falling in the given interval. Only stationary flows are considered here, in which $a(t,\tau)=\eta\tau$ where $\eta=const$ is transition flow rate. The accepted assumptions concerning properties of event flows are usual for applied problems as these flows (or flows which are similar to them) are often occur in practice owing to the relevant limit theorems.

Used for training results interpretation are Markovian processes with discrete states, for which both initial distributions of probabilities and observed frequencies of being in process states $F_{i,d}\left\{F_{i}\right\}_{i=0,\ldots,n-1}$ at time points $\left\{t_{d}\right\}_{d=0,\ldots,D-1}$ are known, where i is state index, D is the number of time points at which frequencies $F_{i,d}$ are available, $0\leq t_{d}\leq T$, T is the terminal time point. Transition flow rates between the model states are fully or partially unknown (free) parameters. As time functions, probabilities of being in the process states are defined by the following set of Kolmogorov ordinary differential equations in matrix form:

$$\frac{d\mathbf{p}\left(t\right)}{dt}=\mathbf{M}(\lambda)\mathbf{p(t)},$$

where $0\leq t\leq T$, $\mathbf{p}\left(t\right)$ represents probabilities of being in the process states, $\mathbf{M}(\lambda)$ is matrix of the transition flow rates between states, λ is the ordered set of these transition flow rates. Continuous-time Markovian processes with the free parameters identified by observation data are referred to hereinafter as *Markovian networks*.

To describe how the probabilities of being in the given states are changed over time Markovian networks organized according to the scheme presented in Figure 1 are applied. This scheme is a finite chain with $2n+2$ states in which transitions from state x_{k} ($k\neq0$, $k\neq n$) is possible to the next state x_{k+1} or state x_{k*} solely. Available from states x_{0} and x_{n} are only

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

states x_1, x_{0*} and x_{n*}, respectively. Being in state x_{k*} ($k=0,...,n$) one can go to state x_k only. For networks of the given type $\mathbf{p}(t_p) = \left(p_0(t), \quad \cdots, \quad p_n(t), p_{0*}(t), \quad \cdots, \quad p_{n*}(t) \right)^T$, $\lambda = \left(\lambda_0^+, ..., \lambda_{n-1}^+, \mu_0^+, ..., \mu_n^+, \mu_0^-, ..., \mu_n^- \right)$, matrix $\mathbf{M}$ has order $2n+2$.

States x_k and x_{k*} correspond to the kth *substantial level* of knowledges, abilities or skills. Its own set of tasks with relevant content is defined for every k. The righter the state, the greater an allowable set of corresponding knowledges, abilities or skills. States with a larger number may (but not need) include the tasks presupposing knowledges, abilities or skills for states with a smaller number. In general case, the complete set of knowledges, abilities or skills corresponds to the rightmost state.
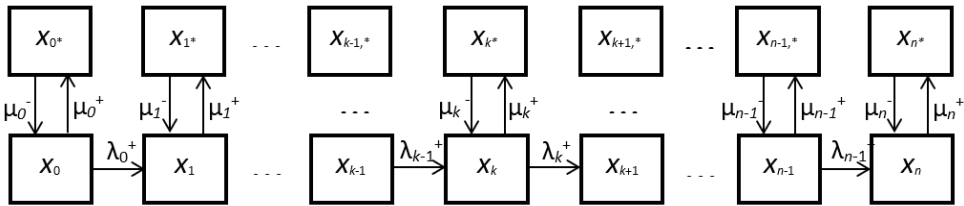

Fig. 1. Continuous-time Markovian network representing the training process: {xi}i=0,...,n and {xi*}i=0,...,n are the states, $\lambda = \left(\lambda_0^+, ..., \lambda_{n-1}^+, \mu_0^+, ..., \mu_n^+, \mu_0^-, ..., \mu_n^- \right)^T$ is the set of transition flow rates between these states

Below, the person who passes a training procedure is referred to as *a learner*. It is assumed that each learner has one of the specified *attainment levels* with indices $i \in \{0,...,z\}$, where $(z+1)$ is number of these levels, with a task kit of a certain difficulty being assigned to each of these levels. Tasks of all attainment levels are assigned to each substantial level of knowledges, abilities or skills.

When a learner is in state x_k the task assigned for him is selected randomly from the task set corresponding to the given state, with time limits being applied for each task.

Transitions between the states are determined by the following rules:
- If being in state x_k a learner performs the assigned task correctly and does not exceed the prescribed time limit, he goes into state x_{k+1}
- If being in state x_k a learner performs the assigned task incorrectly and does not exceed the prescribed time limit, he remains in state x_k
- If being in state x_k and performing the assigned task a learner exceeds the prescribed time limit, he goes into state x_{k*}
- If being in state x_{k*} and performing the assigned task a learner either exceeds the prescribed time limit or performs the assigned task incorrectly and does not exceed the prescribed time limit, he remains in state x_{k*}
- If being in state x_{k*} a learner performs the assigned task correctly and does not exceed the prescribed time limit, he returns into state x_k.

A learner is assumed to be in state x_0 at the initial time point. Upon completing the training procedure under consideration he turns out in one of the states which fit his level of knowledges, abilities or skill best of all. This procedure is terminated when either overall

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

allotted procedure time limit is exceeded or the task assigned in state x_n is completed successfully without reaching the prescribed time limit.

Free parameters of Markovian networks are identified using observed and expected histograms representing frequency distributions of model states, viz.: their estimates are calculated to get the best fit of observed and expected frequencies of being in a certain system's state at the given time points. For this purpose, calculated is a set of transition flow rates λ, which yiel the least value of *the Pearson statistic*:

$$X^2(\lambda) = \sum_{d=0}^{D-1} \sum_{i=0}^{n} \left[\frac{\left(p_i(t_d) N_d - F_{i,d} \right)^2}{p_i(t_d) N_d} + \frac{\left(p_{i*}(t_d) N_d - F_{i*,d} \right)^2}{p_{i*}(t_d) N_d} \right], .$$

where $N_d = \sum_{i=0}^{n} \left(F_{i,d} + F_{i*,d} \right)$. This statistic is used as a model goodness-of-fit measure.

It has been proved that, when a set of common conditions is fulfilled, values of the Pearson statistic, which are obtained by substitution of true solutions, are asymptotically described by χ^2-distribution with $(2n+1)D-l$ degrees of freedom, where l is the number of estimated parameters. Besides, the calculated estimates of free model parameters converge in probability to a required solution with increasing sample size. This fact makes it possible to apply the statistics in question for checking the hypothesis stating that the predicted state hit frequencies fit the observation results.

Identification of continuous-time Markovian models is carried out using data samples consisting of learners' testing outcomes. Each attainment level $i \in \{0, \ldots, z\}$ is processed separately. A unique set of estimates of model parameters λ is associated with each attainment level. It enables further classification by figuring out an attainment level that has the best fit to observations.

To perform the corresponding procedure it is necessary to specify the set of equations $\frac{d\mathbf{p}(t)}{dt} = \mathbf{M}(\lambda)\mathbf{p(t)}$, initial conditions $\mathbf{p}(0)$, initial approximation λ^0, observed frequencies $\left\{ F_{i,d}; F_{i*,d} \right\}_{i=0,\ldots,n}$ of being in the model states, integration step Δt for numerical solution of the given equation set, and accuracy of estimation. A special numerical method has been developed by authors to solve this identification problem [5]. As a result, λ elements which minimize functional $X^2 \lambda$ are determined.

Knowing the model state, in which a tested learner turns out to be after solving the last task at a specified time point, and probabilities of being in this state at the specified time for each attainment level, which can be calculated using Kolmogorov equations, it is possible to estimate posterior probabilities of attainment levels with the aid of *the Bayes formulae*:

$$P(C_i|S) = \frac{P(C_i) P(S|C_i)}{\sum_{k=0}^{z} P(C_k) P(S|C_k)},$$

where C_i is an event indicating that a learner has the ith attainment level ($i \in \{0, \ldots, z\}$), S is an event indicating that a learner is located in the specified model state corresponding to a specified task substantial level at the specified time, $P(C_i)$ is a prior probability for a learner to have the ith attainment level, $P(S|C_i)$ is probability of being in the specified

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

model state at the specified time given a learner has the ith attainment level, $P(C_i|S)$ is probability of the ith attainment level given a learner is located in the specified model state at the specified time.

The attainment level at which the highest conditional probability $P(C_{max}|S) = \max\{P(C_i \mid S)\}_{i=0,\ldots,z}$ is reached yields the required classification. Probability distribution $\{P(C_i \mid S)\}_{i=0,\ldots,z}$ obtained as a result of performing the assigned task sequence makes it possible to estimate the reliability of the derived classification.

Difficulty of a task assigned to a learner corresponds to the current estimate of its attainment level. At the initial time point of a training procedure the task of both the least difficulty level and the least substantial level is assigned. When a learner being in a certain model state has tried to complete yet another task, the abovementioned $z+1$ Bayesian posterior probability estimates of having the ith attainment level are calculated. Herein, only those identified Markovian networks are used, which correspond to the difficulty level of the last assigned task. If it turns out that the most probable attainment level do not correspond to the result obtained after the previous task, the current attainment level is considered as the most probable one for the learner who is forced to get tasks from state x_0 (the "state reset" is executed).

It is desirable to provide some sort of "*inertness*" for such transitions to state x_0, performing them only if the difference between the highest estimated probability and the probability of a current attainment level is greater than a predetermined *threshold value*. At the end of attempts to complete tasks at the least substantial level these "state resets" are not executed.

Hence, a training procedure is reduced to assigning the tasks, successful accomplishment of which requires specified knowledges, abilities or skills in case of a certain attainment level. The formal goal is to bring a learner to the far right state of a Markovian network. This corresponds to mastering all the knowledges, abilities or skills at a certain attainment level. During a training procedure an adaptive principle of task selection is used, according to which difficulties of performed tasks must be in correspondence with the estimations of learners' attainment levels. According to the recent investigations and results of the Item Response Theory, this approach yields the best differentiation of learners by their attainment level.

Both Markovian networks and abovementioned adaptive transitions remain hidden for the learners who have access to assigned tasks only and do not know all the intrinsic mathematical details of a training procedure.

Details of the software implementation as well as results of a pilot experiment to estimate the effectiveness of this adaptive trainer concept are under consideration in paper [4].

3. SELF-LEARNING PROCEDURE

Self-learning procedure for the trainer yields solution of the following problems:
− Optimization of the parameters (transition flow rates) to be identified during collection of results in observing the trainer users;

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

– Development of abilities and skills through the selection of tasks, which contributes to their successful implementation and stimulates the learning process.

To solve the first problem, after collecting sufficient training results and corresponding extension of the training sample, specifying identification of Markovian processes for all attainment levels under consideration is carried out.

To solve the second problem, a square *matrix of successful transitions* $\mathbf{Q}_i = q_{uv,i}$ of order r, where r is the number of tasks used by the trainer, $q_{uv,i}$ is current sample estimation of the probability for successful performing task u providing successful performing task v by the subject of the given attainment level c_i ($i=0,...,z$), is gradually formed for each attainment level under consideration during the training process.

In case of transitions with increase in a substantial level (from state x_k to state x_{k+1}), a subject gets tasks that have a current probability of successful completion exceeding 0.75, if these tasks are available. Tasks that meet this condition are selected randomly. If such tasks are absent, the matrix of successful transitions is not in use. Such organization of task selection promotes the gradual development of skills and abilities by setting before the learners realistically achievable goals in the zone of the nearest development.

4. ASSESSING STATISTICAL SIGNIFICANCE OF THE ADAPTIVE TRAINING EFFECTIVENESS

To assess the effectiveness of the adaptive trainer concept under consideration, its implementation intended for training school students to solve mathematical tasks covered by the school curriculum was employed. There were *three* attainment levels and *four* substantial levels of knowledges, abilities or skills. Both details of this implementation and the information about the supporting software are given in [12, 16, 17, 18].

The experiment, in which two equivalent samples of Moscow penultimate grade school students from two different schools participated, was carried out. Both comparable samples included students with various academic performances. The students from the 1ˢᵗ sample (Group 1 of 24 persons) used the adaptive trainer to prepare themselves for test solution. The students from the 2ⁿᵈ sample (Group 2 of 23 persons) did not use the adaptive trainer at all. After the training all the experiment participants carried out the same test which results were estimated using the 100-point scale. Also evaluated are the greatest quantitative difficulty of a correctly performed task, and the number of correctly performed tasks.

4.1. Univariate Statistical Analysis

The experiments revealed high efficiency of the adaptive trainer, viz.: the given mean test rating has increased 1.54 times owing to its use.

The Mann-Whitney U Test for three observed parameters, viz.: "*The total assessment for performing the mathematical test work on a 100-point scale*", "*The greatest difficulty of a correctly performed task*", and "*The number of correctly performed tasks*", – revealed that:

– There are **significant differences** between Group 1 and Group 2 with regard to the indices "*The total assessment for performing the mathematical test work on a 100-point*

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

scale" and "*The number of correctly performed tasks*" (correspondingly, *Mann-Whitney U Test: 147 and 169, p<0.0055 and p<0.023*)

− There is a **trend** for significant differences between Group 1 and Group 2 with regard to the index "*The greatest difficulty of a correctly performed task*" (*Mann-Whitney U Test: U=190, p<0.069*).

Since the student's samples (Group 1 and Group 2) differ significantly or almost significantly according to given test for all the observed parameters in question, **the revealed effect of adaptive trainer efficiency should be considered as statistically reliable one**.

4.2. Multivariate Statistical Analysis: Structural Equation Modeling

In order to evaluate subtly the level of statistical significance for the adaptive training factor under study, *the structural equation modeling* technique was applied. The relevant factor model, which is represented in the form of path diagrams in Figure 2, takes into account both the effects of the ability to solve mathematical problems, which do not depend on the adaptive training factor, and the effects of the adaptive training factor, as well as the measurement errors, with the assessments for solving the given mathematical tasks being used as observed variables. Two student's groups represented factor structures with and without the adaptive training factor were employed in the path diagrams in use (Group 1 and Group 2 in Figure 2). The assessments under consideration were derived from the results of a mathematics test work that was the same for both student groups.

In the factor model, observed parameters represent three sorts of the test work results in question, viz.: index "*The greatest difficulty of a correctly performed task*" is given as variable A, index "*The number of correctly performed tasks*" is given as variable B, as well as index "*The total assessment for performing the mathematical test work on a 100-point scale*", which is represented by variable Q. In the same model, latent variables P_1 and P_2 represent the factors responsible for the abilities to solve mathematical problems, which do not depend on the adaptive training factor, for the indices "*The greatest difficulty of a correctly performed task*" and "*The number of correctly performed tasks*", respectively; latent variable M represents the adaptive training factor itself; latent variables S_1 and S_2 represent the united factors combining of the aforementioned abilities to solve mathematical problems and the adaptive training effects, if any; latent variables E_1, E_2 and E_3 represent measurement errors for relevant observed variables. Factor loadings a, b, q_1, q_2, m, p_1, p_2, e_1, e_2, e_3 and correlation r are free model parameters to be identified. The sample covariance matrix of observed variables for Group 1 is calculated using the test work results for the students who suffered the adaptive training. Accordingly, the same matrix for Group 2 is obtained for the students who performed the given mathematics test work without the preliminary adaptive training.

Free parameters of the given factor model are identified by the maximum likelihood method, with the following statistics being applied as goodness-of-fit measures to be minimized for both groups simultaneously:

$$F=F_{+}+F_{-},$$
$$F_{+} = [\ln|\mathbf{\Sigma}_{+}| - \ln|\mathbf{C}_{+}| + \mathrm{tr}(\mathbf{S}_{+}\mathbf{\Sigma}_{+}^{-1}) - n]\,(N_{+}-1),$$
$$F_{-} = [\ln|\mathbf{\Sigma}_{-}| - \ln|\mathbf{C}_{-}| + \mathrm{tr}(\mathbf{S}_{-}\mathbf{\Sigma}_{-}^{-1}) - n]\,(N_{-}-1),$$

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
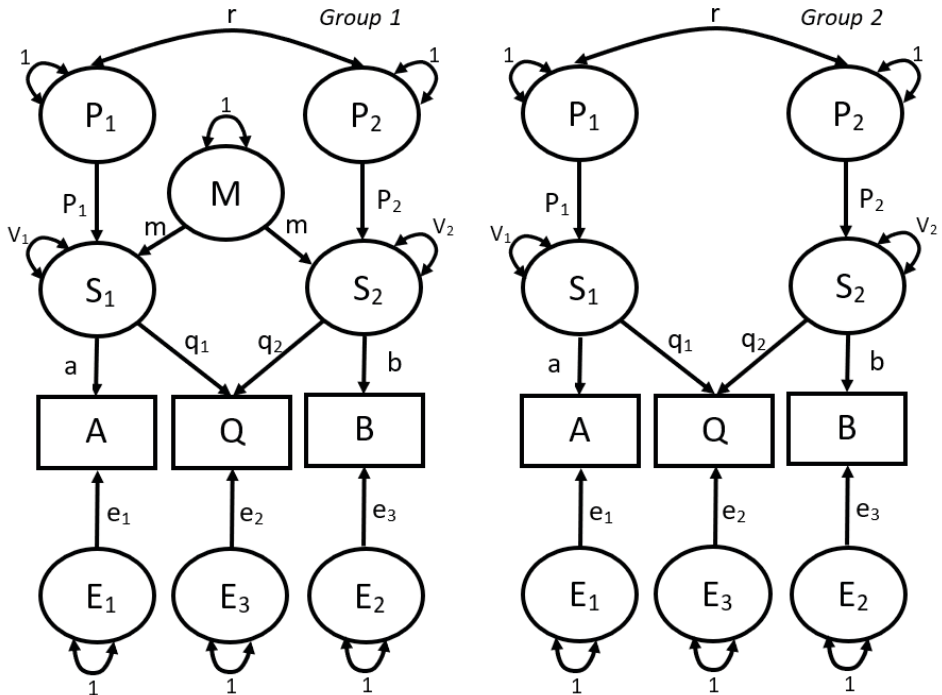*Моделирование и анализ данных. 2021. Том 11. № 4.*

*Fig. 2. Path diagrams: two groups representing the factor structures
with and without the adaptive training factor M*

where F is the maximum likelihood criterion in use, with F_+ and F_- being its group components; C_+ and C_- are the sample covariance matrices of observed variables for Group 1 and Group 2, respectively; Σ_+ and Σ_- – the corresponding expected covariance matrices of observed variables; $|\Sigma_+|$, $|\Sigma_-|$, $|C_+|$ and $|C_-|$ are determinants of relevant matrices; tr – trace of a relevant matrix; N_+ and N_- are the sample sizes used to calculate matrices C_+ and C_-, respectively; n is number of observed variables in a group ($n=3$). Expected covariance matrices for the groups under consideration are expressed via analytical expressions composed of free parameters and have the following form (observed parameters are given in the following order: A, B, Q):

$$\Sigma_+ = \begin{pmatrix} a^2(p_1^2 + m^2) + e_1^2 & & symmetrically \\ abrp_1p_2 + abm^2 & b^2(p_2^2 + m^2) + e_3^2 & \\ ap_1(p_1q_1 + rp_2q_2) + am^2(q_1 + q_2) & bp_2(p_2q_2 + rp_1q_1) + bm^2(q_1 + q_2) & q_1^2p_1^2 + q_2^2p_2^2 + 2rq_1p_1q_2p_2 + m^2(q_1 + q_2)^2 + e_2^2 \end{pmatrix},$$

$$\Sigma_- = \begin{pmatrix} a^2p_1^2 + e_1^2 & & symmetrically \\ abrp_1p_2 & b^2p_2^2 + e_3^2 & \\ ap_1(p_1q_1 + rp_2q_2) & bp_2(p_2q_2 + rp_1q_1) & q_1^2p_1^2 + q_2^2p_2^2 + 2rq_1p_1q_2p_2 + e_2^2 \end{pmatrix}.$$

Under the assumption of multivariate normal distribution of the observed variables under study the values of statistics F are described by the χ^2 distribution.

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

Multidimensional optimization problem should be solved numerically to identify free model parameters.

The initial saturated model under consideration has 3 observed variables, 12 independent sample statistics, 11 free model parameters and, correspondingly, 1 degree of freedom for the χ^2 distribution of F-statistics. Initial model optimization by reducing the number of free parameters results in $e_1=e_3=0$ with non-significant changes in the maximum likelihood criterion F and 3 degrees of freedom in the obtained new initial saturated model.

To assess statistical significance of the motion factor under study, the saturated model presented in Figure 2 should be compared with its reduced option without latent variable M. The difference in F-statistics between the saturated and reduced models is asymptotically distributed as χ^2, with number of degrees of freedom being equal to the difference in their numbers of degrees of freedom.

Identification of the obtained optimized initial saturated initial model yields acceptable fit for the observed and expected covariance matrices in use ($F=7.53$, $df=3$, $p<0.06$) with non-zero values for factor loading m. Comparing changes in F-statistics resulted from identifying the saturated and reduced model shows statistically significant differences attributable to effects caused by adaptive training factor M ($\Delta F=7.38$, $df=1$, $p<0.007$). So, ***it must be concluded that there are statistically significant influences of the adaptive training factor on the observed indices "The greatest difficulty of a correctly performed task" and "The number of correctly performed tasks" as well as on the total assessment quantity "The total assessment for performing the mathematical test work on a 100-point scale".***

4.3. Multivariate Statistical Analysis: Discriminant Analysis

In addition, ***the Fischer's discriminant analysis of the given indices revealed that these characteristics in total contain the information capable of dividing Group 1 and Group 2.*** Corresponding results are presented in Table 1.

Table 1

Indices "The total assessment for performing the mathematical test work on a 100-point scale", "The greatest difficulty of a correctly performed task", and "The number of correctly performed tasks": recognition results

	% of correct recognition (Wilks' Lambda: 0.63; approx. F (3,43)=8.18; p< .0002: acceptable discrimination)	Group 1	Group 2
Group 1	70	17	7
Group 2	69	7	16
Total	**70** (14 errors of 47 cases)	24	23

Generally, it should be also noted that the results obtained with the aid of the approaches under consideration do not contradict each other, with the final conclusions being reinforced.

5. PRINCIPAL RESULTS

1. A concept of the adaptive trainer that provides selection of tasks using parametric mathematical models represented by the discrete-state continuous-time Markovian random processes has been developed.

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

2. Free parameters of the Markovian processes used to describe the adaptive trainer are identified with the aid of observed and expected histograms representing frequencies of being in the model states. This identification is carried out separately for each of the attainment levels under consideration.

3. Affiliation with different levels of training is determined with the aid of the Bayesian estimates.

4. Features of the diagnostic techniques used to select the tasks are:
   – Detecting and using temporal dynamics of variable subject ability to cope with the test tasks when constructing the calculated estimates
   – Possibility to take into account the time spent for performing tasks when constructing the calculated estimates
   – In comparison with other approaches, fewer number of presented tasks to speed up the testing process
   – Advanced technique for identification of model parameters.

   These capabilities ensure advantages of the presented approach over similar ones.

5. Self-learning procedure for the trainer yields solutions of both optimization problem for the identified parameters and learner's development by means of selection which promotes tasks to be performed successfully. Using the matrix of successful transitions to select tasks for the learners results in the gradual development of their skills and abilities by setting realistically achievable goals in the zone of the nearest development.

6. The experiments revealed high efficiency of the adaptive trainer, viz.: the mean test rating has increased 1.54 times owing to its use. The revealed effect of adaptive trainer efficiency for the observed test results should be considered as statistically reliable one.

7. The structural equation modeling revealed statistically significant influences of the adaptive training factor on the observed indices "The greatest difficulty of a correctly performed task" and "The number of correctly performed tasks" as well as on the total assessment quantity "The total assessment for performing the mathematical test work on a 100-point scale" in total.

8. The Fischer's discriminant analysis of the observed indices "The greatest difficulty of a correctly performed task", "The number of correctly performed tasks", and "The total assessment for performing the mathematical test work on a 100-point scale" revealed that these characteristics in total contain the information capable of dividing the samples of trained and untrained students.

References

1. Baker F.B. The Basics of Item Response Theory. – ERIC Clearinghouse on Assessment and Evaluation, University of Maryland, College Park, MD. 2001.
2. Ebmodo. Connect with Students and Parents in Your Paperless Classroom. URL: https://www.edmodo.com.
3. Kats, Y. Learning Management Systems and Instructional Design: Best Practices in Online Education. – IGI Global. 2013. – ISBN 9781466639317
4. Kuravsky L.S., Margolis A.A., Marmalyuk P.A., Panfilova A.S., Yuryev G.A. Mathematical Aspects of the Adaptive Trainer Concept. – Psikhologicheskaya Nauka i Obrazovanie (Psychological Science and Education), 2016, Vol. 21, No. 2, pp. 84–95, doi: 10.17759/pse2016210210.

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

5. Kuravsky L.S., Marmalyuk P.A., Yuryev G.A. and Dumin P.N. A Numerical Technique for the Identification of Discrete-State Continuous-Time Markov Models// Applied Mathematical Sciences. Vol. 9, 2015, No. 8, pp. 379–391. URL: http://dx.doi.org/10.12988/ams. 2015.410882.

6. Kuravsky L.S., Marmalyuk P.A., Yuryev G.A., Dumin P.N. and Panfilova A.S. Probabilistic Modeling of a Testing Procedure // Applied Mathematical Sciences. Vol. 9, 2015, No. 82, pp. 4053–4066. URL: http://dx.doi.org/10.12988/ams.2015.53234.

7. Kuravsky L.S., Artemenkov S.L., Yuryev G.A., Grigorenko E.L. A new approach to computerized adaptive testing. – Experimental Psychology, 2017, Vol. 10, No 3, pp. 33–45, http://dx.doi.org/10.17759/exppsy.2017100303.

8. Kuravsky L.S., Margolis A.A., Marmalyuk P.A., Yuryev G.A., Dumin P.N. Trained Markov Models to Optimize the Order of Tasks in Psychological Testing. – Neirokompyutery: Razrabotka, Primenenie (Neurocomputers: Development, Application), 2013, No. 4, pp. 28–38 (in Russian).

9. Kuravsky L.S., Yuriev G.A. Probabilistic method of filtration artifacts in adaptive testing. – Experimental Psychology, Vol.5, No.1, 2012, p. 119–131 (in Russian).

10. Kuravsky L.S., Yuryev G.A. Using Markov Models in Test Results Processing. – Voprosy Psychologii (Problems of Psychology), 2011, No. 2, pp. 98–107 (in Russian).

11. Kuravsky L.S., Marmalyuk P.A., Yuryev G.A., Belyaeva O.B. and Prokopieva O.Yu. Mathematical Foundations of Flight Crew Diagnostics Based on Videooculography Data. – Applied Mathematical Sciences, 2016, Vol. 10, No. 30, pp. 1449–1466, http://dx.doi.org/10.12988/ams.2016.6122.

12. Markov models in the diagnostics and prediction problems: Textbook. / Edited by L.S. Kuravsky. – 2nd Edition, Enlarged. – M.: MSUPE Edition, 2017. – 203 pp. (in Russian).

13. Moodle Open-Source Learning Platform// Moodle Pty Ltd. URL:https://moodle.org.

14. Rasch, G. Probabilistic Models for Some Intelligence and Attainment Tests // Copenhagen, Danish Institute for Educational Research, Expanded Edition (1980). Chicago: The University of Chicago Press. 1960/1980.

15. Wright B.D., Masters G.N. Rating Scale Analysis. Rasch Measurements // Chicago: MESA Press. 1982. 206 pp.

16. Pominov D.A., Kuravsky L.S., Dumin P.N., Yuriev G.A. Adaptive trainer for preparing students for mathematical exams. – International Journal of Advanced Research in Engineering and Technology, 2020, Vol. 11, No 11, pp. 260–268, doi 10.34218/IJARET.11.11.2020.022.

17. Pominov D.A. Adaptive trainer for preparing students for math exams. – Neirokompyutery: Razrabotka, Primenenie (Neurocomputers: Development, Application), 2021, Vol. 23, No. 2, pp. 35–42 (in Russian).

18. Pominov D.A. Model of adaptive learning and his implementation. – Modelirovaniye i analiz dannykh (Modelling and Data Analysis), 2020, Vol. 10, No 3, pp. 39–52 (in Russian).

19. Margolis A.A., Safronova M.A., Khaperskaya A.U. The view from Russia: Overcoming students' learning challenges. Research Intelligence, 2021, issue, pp. 30–31, https://www.bera.ac.uk/publication/autumn-2021.

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

Концепция адаптивного тренажера и оценка его эффективности в математическом обучении

*Куравский Л.С.**

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/0000-0002-3375-8446
e-mail: l.s.kuravsky@gmail.com

*Поминов Д.А.***

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/0000-0002-1321-3713
e-mail: pominovda@mgppu.ru

*Юрьев Г.А. ****

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/ 0000-0002-2960-6562
e-mail: g.a.yuryev@gmail.com

*Юрьева Н.Е.*****

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/0000-0003-1419-876X
e-mail: yurieva.ne@gmail.com

*Сафронова М.А.******

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/0000-0002-3597-6375
e-mail: mariasaf@gmail.com

*Куланин Е.Д.*******

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/0000-0001-6093-7012
e-mail: lucas03@mail.ru

*Антипова С.Н.********

Московский государственный психолого-педагогический университет,
г. Москва, Российская Федерация
ORCID: https://orcid.org/0000-0001-6642-7953
e-mail: antipovasn@mgppu.ru

Представлена математическая модель самообучающегося адаптивного тренажера. Применяемый подход является альтернативой адаптивным технологиям, основанным на современной теории тестирования (англ. Item Response Theory). Его особенностями являются учёт временной динамики

Kuravsky L.S., Pominov D.A., Yuryev G.A., Yuryeva N.E., Safronova M.A., Kulanin Y.D., Antipova S.N.
The Concept of an Adaptive Trainer and Assessing Its Effectiveness in a Mathematical Application
Modelling and Data Analysis 2021. Vol. 11, no. 4.

адаптивной процедуры обучения и меньшее количество заданий, которые следует выполнить для обеспечения намеченного результата. Для оценки эффективности данной концепции использовалась веб-реализация тренажера, предназначенная для обучения решению математических задач в рамках школьной программы. Проведенный анализ выявил высокую эффективность и статистически значимое влияние фактора адаптивного обучения на результаты выполнения контрольного теста.

Ключевые слова: адаптивное обучение, марковские случайные процессы, адаптивный тренажер, самообучающиеся системы.

Куравский Лев Семенович, доктор технических наук, декан факультета информационных технологий, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/0000-0002-3375-8446, e-mail: l.s.kuravsky@gmail.com

****Поминов Денис Александрович,*** младший научный сотрудник, факультет информационных технологий, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/0000-0002-1321-3713, e-mail: pominovda@mgppu.ru

*****Юрьев Григорий Александрович,*** кандидат физико-математических наук, доцент, заведующий научной лабораторией, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/ 0000-0002-2960-6562, e-mail: g.a.yuryev@gmail.com

******Юрьева Наталия Евгеньевна,*** кандидат технических наук, научный сотрудник, центра информационных технологий для психологических исследований факультета информационных технологий, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/0000-0003-1419-876X, e-mail: yurieva.ne@gmail.com

*******Сафронова Мария Александровна,*** кандидат психологических наук, декан факультета Психология образования, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/0000-0002-3597-6375, e-mail: mariasaf@gmail.com

********Куланин Евгений Дмитриевич,*** кандидат физико-математических наук, профессор, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/0000-0001-6093-7012, e-mail: lucas03@mail.ru

*********Антипова Светлана Николаевна,*** заместитель декана по внеучебной работе факультета информационных технологий, Московский государственный психолого-педагогический университет, г. Москва, Российская Федерация, ORCID: https://orcid.org/0000-0001-6642-7953, e-mail: antipovasn@mgppu.ru

Куравский Л.С., Поминов Д.А., Юрьев Г.А., Юрьева Н.Е., Сафронова М.А., Куланин Е.Д., Антипова С.Н.
Концепция адаптивного тренажера и оценка его эффективности в математическом обучении
Моделирование и анализ данных. 2021. Том 11. № 4.

Литература

1. *Baker F.B.* The Basics of Item Response Theory. – ERIC Clearinghouse on Assessment and Evaluation, University of Maryland, College Park, MD. 2001.

2. Ebmodo. Connect with Students and Parents in Your Paperless Classroom. URL: https://www.edmodo.com.

3. *Kats, Y.* Learning Management Systems and Instructional Design: Best Practices in Online Education. – IGI Global. 2013. – ISBN 9781466639317

4. *Куравский Л.С., Марголис А.А., Мармалюк П.А., Панфилова А.С., Юрьев Г.А.* Математические аспекты концепции адаптивного тренажера // Психологическая наука и образование. 2016. Том 21. № 2. С. 84–95. doi:10.17759/pse.2016210210

5. *Kuravsky L.S., Marmalyuk P.A., Yuryev G.A. and Dumin P.N.* A Numerical Technique for the Identification of Discrete-State Continuous-Time Markov Models // Applied Mathematical Sciences. Vol. 9, 2015, No. 8, pp. 379–391. URL: http://dx.doi.org/10.12988/ams. 2015.410882.

6. *Kuravsky L.S., Marmalyuk P.A., Yuryev G.A., Dumin P.N. and Panfilova A.S.* Probabilistic Modeling of a Testing Procedure // Applied Mathematical Sciences. Vol. 9, 2015, No. 82, pp. 4053–4066. URL: http://dx.doi.org/10.12988/ams.2015.53234.

7. *Kuravsky L.S., Artemenkov S.L., Yuryev G.A., Grigorenko E.L.* A new approach to computerized adaptive testing. – Experimental Psychology, 2017, Vol. 10, No 3, pp. 33–45, http://dx.doi.org/10.17759/exppsy.2017100303.

8. *Куравский Л.С., Марголис А.А., Мармалюк П.А., Юрьев Г.А., Думин П.Н.* Обучаемые марковские модели в задачах оптимизации порядка предъявления психологических тестов // Нейрокомпьютеры: разработка, применение. 2013. №4. С.28–38.

9. *Kuravsky L.S., Yuriev G.A.* Probabilistic method of filtration artifacts in adaptive testing. – Experimental Psychology, Vol.5, No.1, 2012, p. 119–131 (in Russian).

10. *Куравский Л.С., Юрьев Г.А.* Использование марковских моделей при обработке результатов тестирования // Вопросы психологии. 2011 №2. С. 98-107/

11. *Kuravsky L.S., Marmalyuk P.A., Yuryev G.A., Belyaeva O.B. and Prokopieva O.Yu.* Mathematical Foundations of Flight Crew Diagnostics Based on Videooculography Data. – Applied Mathematical Sciences, 2016, Vol. 10, No. 30, pp. 1449–1466, http://dx.doi.org/10.12988/ams.2016.6122.

12. Марковские модели в задачах диагностики и прогнозирования: Учеб. пособие. / Под ред. Л.С. Куравского. – 2-е изд., доп. – М.: Изд-во МГППУ, 2017. – 203 с.: ил. – ISBN 978–5–94051–168–7.

13. Moodle Open-Source Learning Platform// Moodle Pty Ltd. URL:https://moodle.org.

14. *Rasch, G.* Probabilistic Models for Some Intelligence and Attainment Tests // Copenhagen, Danish Institute for Educational Research, Expanded Edition (1980). Chicago: The University of Chicago Press. 1960/1980.

15. *Wright B.D., Masters G.N.* Rating Scale Analysis. Rasch Measurements // Chicago: MESA Press. 1982. 206 pp.

16. *Pominov D.A., Kuravsky L.S., Dumin P.N., Yuriev G.A.* Adaptive trainer for preparing students for mathematical exams. – International Journal of Advanced Research in Engineering and Technology, 2020, Vol. 11, No 11, pp. 260–268, doi 10.34218/IJARET.11.11.2020.022.

17. *Поминов Д.А.* Самообучающийся адаптивный тренажер для подготовки абитуриентов к экзаменам по математике // Нейрокомпьютеры: разработка, применение. 2021. Т. 23. № 2. С. 35–42. – DOI 10.18127/j19998554-202102-04

18. *Поминов Д.А.* Модель процесса адаптивного обучения и его программная реализация // Моделирование и анализ данных. 2020. Том 10. № 3. С. 39–52.

19. *Margolis A.A., Safronova M.A., Khaperskaya A.U.* The view from Russia: Overcoming students' learning challenges. Research Intelligence, 2021, issue, pp. 30–31, https://www.bera.ac.uk/publication/autumn-2021.